INTEGRATION BY PARTS

According to integration by parts if u and v ($f(x)$ and $g(x)$) are two differentiable function of a single ~~variable~~ independent variable x. Then,

$$\int f(x).g(x).dx = f(x).\int g(x) - \int f'(x).\int g(x)$$

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The integral of the product of two functions = (first function × integration of second function) − integration of [(differential of first function) × integration of second function]

QUESTIONS

Q) Integrate $\int x.\sin x.dx$

Solution, Given $y = \int x.\sin x.dx$

Let $I_1 = \cancel{x \int \cos x}$ $I_1 = x$, $I_2 = \sin x$.

Now, using ILATE we get

$x.(-\cos x) - \int 1.-\cos x$

$-x\cos x + \sin x$

$\therefore \sin x - x\cos x + C$ Ans

Q) Evaluate $\int x^2.e^{mx}\,dx$

Solution:- Given

$y = \int x^2 e^{mx}$

$I_1 = x^2, I_2 = e^{mx}$

Using integration by parts we get-

$$y = x^2 \cdot \frac{e^{mx}}{m} - \int 2x \cdot \frac{e^{mx}}{m}$$

$$= \frac{x^2}{m} e^{mx} - \frac{2}{m} \int x \cdot e^{mx}$$

$$= \text{"} \quad \text{"} - \frac{2}{m}\left[x \cdot \frac{e^{mx}}{m} - 1 \cdot \frac{e^{mx}}{m^2}\right]$$

$$= \frac{x^2}{m} e^{mx} - \frac{2 \cdot x \cdot e^{mx}}{m^2} + \frac{2}{m^3} e^{mx}$$

$$y = \frac{e^{mx}}{m}\left[x^2 - \frac{2x}{m} + \frac{2}{m^2}\right] \text{ Ans}$$

Q) Find the value of $\int \log x \, dx$

Solution: Given, $y = \int \log x \cdot 1$

Let- $I_1 = \log x$

$I_2 = 1$

Then using ILATE we get

$$y = \log x \cdot x - \int \frac{1}{x} \cdot x$$

$$= \log x \cdot x - x + c$$

$$= x(\log x - 1) \text{ Ans}$$

Q9) Evaluate $\int \sin^{-1} x \, dx$.

Solution. $y = \int \sin^{-1} x \, dx = \int \sin^{-1} x \cdot 1$

Let $I_1 = \sin^{-1} x$

$I_2 = 1$

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Now, using ILATE

$= \sin^{-1}x \cdot x - \int \frac{1}{\sqrt{1-x^2}}\, x$

$\sin^{-1}x \cdot x -$

let $1 - x^2 = t$

$-dx = \frac{dt}{dx}$

$\therefore \sin^{-1}x \cdot x - \int -\frac{x}{\sqrt{t}} \frac{dt}{dx}$

$\Rightarrow \quad " \quad + \int \frac{dt}{\sqrt{t}}$

$\Rightarrow \quad " \quad + \cancel{2}\sqrt{(1-x^2)} + C$

$\therefore x \sin^{-1}x + \sqrt{(1-x^2)} + C$ Ans

Q.) Integrate: $\int x^3 \tan^{-1}x \, dx$

solution, $y = \int x^3 \tan^{-1}x \, dx$

Now, using ILATE we get

$\tan^{-1}x \cdot \frac{x^2}{2} - \int \frac{1}{(1+x^2)} x^2$

$= \tan^{-1}x \cdot \frac{x^2}{2} - \int \frac{(x^2)^2}{(1+x^2)}$

~~let $1 + x^4 = 2t$~~

~~$dx = \frac{dt}{dx}$~~

$+ \frac{(t-1)^2}{t}$

$= \tan^{-1}x \cdot \frac{x^2}{4} - \frac{1}{4}\int \frac{x^4}{(1+x^2)}$

By dividing diff

$= \tan^{-1}x \cdot \frac{x^2}{4} - \frac{1}{4}\int (x^2-1) + \frac{1}{(x^2+1)}$

$= \tan^{-1}x \cdot \frac{x^2}{4} - \frac{1}{4}\left[\left(\frac{x^3}{3} - x\right) + \tan^{-1}x\right]$

$= \tan^{-1}x \cdot \frac{x^4}{4} - \frac{1}{12}x^3 + \frac{x}{4} - \frac{1}{4}\tan^{-1}x$

$= \tan^{-1}x \, \frac{1}{4}(x^4 - 1) - \frac{1}{12}x^3 + \frac{x}{4} + C$